

POLARIZED FRAGMENTATION FUNCTIONS

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In this talk I present a review on the theoretical status of polarized fragmentation functions and the prospects for conceivable future semi-inclusive deep-inelastic scattering and proton-proton collision experiments to measure them.

1 Introduction

Measurements of rates for single-inclusive e^+e^- annihilation (SIA) into a specific hadron H ,

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow H X \quad , \quad (1)$$

play a similarly fundamental role as those of the corresponding crossed ‘space-like’ deep-inelastic scattering (DIS) process $ep \rightarrow e'X$. Their interpretation in terms of scale-dependent fragmentation functions $D_f^H(z, Q^2)$, the ‘time-like’ counterparts of the parton distribution functions $f_H(x, Q^2)$ of a hadron H , hence provides a further important, complementary test of perturbative QCD. Furthermore, in the polarized case, they provide information about how the spin carried by the partons is passed to the hadrons in the fragmentation process and, at variance with DIS, it is possible to measure them even for the case of hadrons which are not available as targets.

Among all the hadrons, the production of Λ baryons appears to be particularly interesting: the self-analyzing properties of its dominant weak decay $\Lambda \rightarrow p\pi^-$ and the particularly large asymmetry of the angular distribution of the decay proton in the Λ rest-frame allow for an experimental reconstruction of the Λ spin.

In the following we will address the issue of fragmentation into polarized Λ ’s within a detailed QCD analysis. We will provide some realistic sets of unpolarized and polarized fragmentation functions for Λ baryons. Since there are hardly any data sensitive to the polarized fragmentation functions ΔD_f^Λ , one has to rely on reasonable assumptions.

Our various proposed sets for ΔD_f^Λ , which are all compatible with the LEP data, are particularly suited for estimating the physics potential of different

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processes to determine the polarized fragmentation functions more precisely. We hence present detailed predictions for Semi-Inclusive DIS (SIDIS) and pp collisions, for both longitudinally and transversely polarized beams and Λ 's. For further details we refer the reader to the original refs.^{1,2,3}.

2 Unpolarized Λ Fragmentation Functions

The cross section for the inclusive production of a hadron with energy E_H in SIA at a c.m.s. energy \sqrt{s} , integrated over the production angle, can be written in the following way⁴:

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^H}{dx_E} = \frac{1}{\sum_q e_q^2} [2 F_1^H(x_E, Q^2) + F_L^H(x_E, Q^2)] , \quad (2)$$

where $x_E = 2p_H \cdot k / Q^2 = 2E_H / \sqrt{s}$ (k being the momentum of the intermediate boson, $k^2 = Q^2 = s$) and σ_{tot} is the total cross section for $e^+e^- \rightarrow hadrons$. The sum in (2), runs over the n_f active quark flavors q , and the e_q are the corresponding appropriate electroweak charges.

The unpolarized ‘time-like’ structure function F_1^H is in LO related to the fragmentation functions $D_f^H(z, Q^2)$ by

$$2 F_1^H(x_E, Q^2) = \sum_q e_q^2 [D_q^H(x_E, Q^2) + D_{\bar{q}}^H(x_E, Q^2)] , \quad (3)$$

the $D_f^H(z, Q^2)$ obeying LO Altarelli-Parisi-type Q^2 -evolution equations. The corresponding NLO expressions for F_1^H and F_L^H which include the relevant NLO ($\overline{\text{MS}}$) coefficient functions are too lengthy to be given here but can be found in⁴. In the last few years several experiments⁵ have reported measurements of the unpolarized cross section for the production of Λ baryons, which allows an extraction of the unpolarized Λ fragmentation functions required for constructing the polarization asymmetries and as reference distributions in the positivity constraint for the polarized ones. We emphasize at this point that the wide range of c.m.s. energies covered by the data ($14 \leq \sqrt{s} \leq 91.2$ GeV) makes a detailed QCD analysis that includes the Q^2 -evolution of the fragmentation functions mandatory.

Unless stated otherwise, we will refer to both Λ^0 and $\bar{\Lambda}^0$ as simply ‘ Λ ’. As a result, the obtained fragmentation functions always correspond to the sum

$$D_f^\Lambda(x_E, Q^2) \equiv D_f^{\Lambda^0}(x_E, Q^2) + D_f^{\bar{\Lambda}^0}(x_E, Q^2) . \quad (4)$$

Since no precise SIDIS data are available yet, it is not possible to obtain individual distributions for all the light flavors separately, and hence some

sensible assumptions concerning them have to be made. Employing naive quark model $SU_f(3)$ arguments and neglecting any mass differences between u, d, s , we *assume* that all the light flavors fragment equally into Λ , i.e.

$$D_u^\Lambda = D_d^\Lambda = D_s^\Lambda = D_{\bar{u}}^\Lambda = D_{\bar{d}}^\Lambda = D_{\bar{s}}^\Lambda \equiv D_q^\Lambda . \quad (5)$$

At the rather low initial scale μ^6 ($\mu_{LO}^2 = 0.23 \text{ GeV}^2$, $\mu_{NLO}^2 = 0.34 \text{ GeV}^2$) we choose the following simple ansatz:

$$z D_f^\Lambda(z, \mu^2) = N_f z^{\alpha_f} (1 - z)^{\beta_f} , \quad (6)$$

where $f = u, d, s$, and g . For details on the treatment of the heavy flavor contributions, we refer to ¹. Utilizing eq. (5) a total of 10 free parameters remains to be fixed from a fit to the available 103 data points ⁵ (we include only data with $x_E > 0.1$ in our fit to avoid possibly large non-perturbative contributions induced by finite-mass corrections). The total χ^2 values are 103.55 and 104.29 in NLO and LO, respectively, and the optimal parameters in (6) can be found in ¹.

A comparison of our LO and NLO results with the data is presented in fig. 1, where all the existing data ⁵ have been converted to the ‘format’ of eq.(2). One should note that the LO and the NLO results are almost indistinguishable, demonstrating the perturbative stability of the process considered. Furthermore, there is an excellent agreement between the predictions of our fits and the data even in the small- x_E region which has not been included in our analysis.

3 Polarized Fragmentation Functions

Having obtained a reliable set of unpolarized fragmentation functions we now turn to the polarized case where unfortunately only scarce and far less precise data are available. In fact, no data at all have been obtained so far using polarized beams. The only available information comes from *unpolarized* LEP measurements profiting from the parity-violating electroweak $q\bar{q}Z$ coupling.

For such measurements, done at the mass of the Z boson (‘ Z -resonance’), the asymmetry is given by

$$A^H = \frac{g_3^H + g_L^H/2}{F_1^H + F_L^H/2} . \quad (7)$$

The polarized *non-singlet* structure function g_3^H is given at LO by

$$g_3^H(x_E, Q^2) = \sum_q g_q [\Delta D_q^H(x_E, Q^2) - \Delta D_{\bar{q}}^H(x_E, Q^2)] , \quad (8)$$

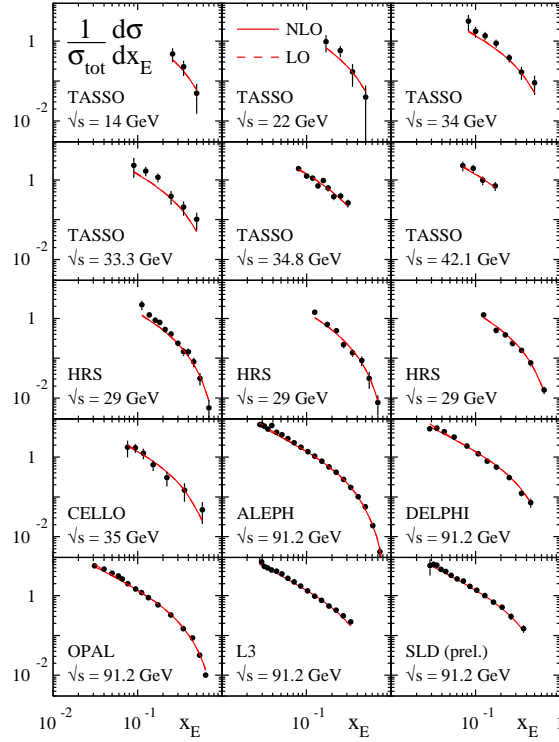


Figure 1: Comparison of LO and NLO results with all available data on unpolarized Λ production in e^+e^- annihilation. Note that only data points with $x_E \geq 0.1$ have been included in our fit.

where the g_q are the appropriate effective charges (see, e.g.,¹). The complete NLO QCD corrections can be found in^{1,7}. Note that only the *valence* part of the polarized fragmentation functions can be obtained from the available LEP data⁵, and that Λ^0 's and $\bar{\Lambda}^0$'s give contributions of opposite signs to the measured polarization and thus to g_3^Λ . Unfortunately, it turns out that with the available LEP data⁵, all obtained on the Z -resonance, it is not even possible to obtain the valence distributions for all the flavors, so some assumptions have to be made here. Obviously, even further assumptions are needed for the polarized gluon and sea fragmentation functions in order to have a complete set of fragmentation functions suitable for predictions for other processes.

The heavy flavor contributions to polarized Λ production are neglected, and u and d fragmentation functions are taken to be equal in this analysis. Furthermore, polarized unfavored distributions, i.e. $\Delta D_u^{\Lambda^0} = \Delta D_u^{\bar{\Lambda}^0}$, etc., and

the gluon fragmentation function ΔD_g^Λ are assumed to be negligible at the initial scale μ . The remaining fragmentation functions are parameterized in the following simple way

$$\Delta D_s^\Lambda(z, \mu^2) = z^\alpha D_s^\Lambda(z, \mu^2) \quad , \quad \Delta D_u^\Lambda(z, \mu^2) = \Delta D_d^\Lambda(z, \mu^2) = N_u \Delta D_s^\Lambda(z, \mu^2) \quad (9)$$

and are subject to positivity constraints. These input distributions are then evolved to higher Q^2 via the appropriate Altarelli-Parisi equations⁸.

Within this framework we try three different scenarios for the polarized fragmentation functions at our low initial scale μ to cover a rather wide range of plausible models:

Scenario 1 corresponds to the expectations from the non-relativistic naive quark model where only s -quarks can contribute to the fragmentation processes that eventually yield a polarized Λ , even if the Λ is formed via the decay of a heavier hyperon. We hence have $N_u = 0$ in (9) for this case.

Scenario 2 is based on estimates by Burkardt and Jaffe^{9,10} for the DIS structure function g_1^Λ of the Λ , predicting sizeable negative contributions from u and d quarks to g_1^Λ by analogy with the breaking of the Ellis-Jaffe sum rule for the proton's g_1^p . Assuming that such features also carry over to the ‘time-like’ case¹⁰, we simply impose $N_u = -0.20$ (see also¹¹).

Scenario 3: All the polarized fragmentation functions are assumed to be equal here, i.e. $N_u = 1$, contrary to the expectation of the non-relativistic quark model used in scen. 1. This rather ‘extreme’ scenario might be realistic if, e.g., there are sizeable contributions to polarized Λ production from decays of heavier hyperons who have inherited the polarization of originally produced u and d quarks.

Our results for the asymmetry A^Λ in (7) within the three different scenarios are compared to the available LEP data⁵ in fig. 2. The optimal parameters in (9) for the three models can be found in¹. As can be seen, the best agreement with the data is obtained within the (naively) most unlikely scen. 3. The differences occur mainly in the region of large x_E , where scen. 1 and 2 cannot fully account for the rather large observed polarization due to the positivity constraints. For instance, in the case of scen. 1, the asymmetry behaves asymptotically roughly like $-\Delta D_s^\Lambda/3D_s^\Lambda$, and even when saturating the positivity constraint at around $x_E = 0.5$ it is not possible to obtain a polarization as large as the one required by the LEP data. Of course, such an argument still depends strongly on the assumed $SU(3)_f$ symmetry for the *unpolarized* fragmentation functions, which could be broken. The situation concerning the Λ fragmentation functions can only be further clarified by future precise measurements for unpolarized *and* polarized beams.

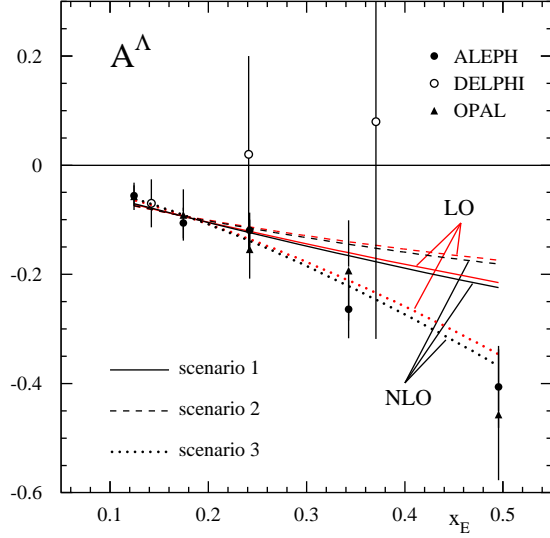


Figure 2: Comparison of LEP data and our LO and NLO results for the asymmetry A^Λ , using the three different scenarios.

Equipped with various sets of polarized fragmentation functions, let us now turn to other processes that might further probe our distributions.

4 SIDIS

The SIDIS process $eN \rightarrow e'HX$ ^b is very well suited to give information on fragmentation functions. In this case, the cross section is proportional to a combination of both the parton distributions of the nucleon N and the fragmentation functions for the hadron H (in this work, we will only refer to the current fragmentation region).

In the particular case where both nucleon and hadron are unpolarized, the cross section can be written in a way similar to the fully inclusive DIS case:

$$\frac{d\sigma^H}{dx dy dz} = \frac{4\pi\alpha^2 xs}{Q^4} \left[(1 + (1-y)^2) F_1^{N/H}(x, z, Q^2) + \frac{(1-y)}{x} F_L^{N/H}(x, z, Q^2) \right] \quad (10)$$

^bThe case of SIDIS with a neutrino beam has been studied in¹².

with the structure function $F_1^{N/H}$ given at LO by

$$2 F_1^{N/H}(x, z, Q^2) = \sum_{q, \bar{q}} e_q^2 q(x, Q^2) D_q^H(z, Q^2) . \quad (11)$$

The corresponding NLO corrections can be found in ^{13,4}. Three other possible cross sections can be defined when the polarization of the lepton, the initial nucleon and the hadron are taken into account. If both nucleon and hadron are polarized and the lepton is unpolarized, the expression is similar to eqs. (10),(11) above with, however, the unpolarized parton distributions *and* the fragmentation functions to be replaced by their polarized counterparts. In the case that the lepton and either the nucleon or the hadron are polarized, the expression for the cross section is given as in the fully inclusive case by a single structure function $g_1^{N/H}(x, z, Q^2)$:

$$\frac{d\Delta\sigma^H}{dx dy dz} = \frac{8\pi\alpha^2 xys}{Q^4} \left[(2-y) g_1^{N/H}(x, z, Q^2) \right] . \quad (12)$$

Here the polarized structure function $g_1^{N/H}$ can be written as ^{14,15}

$$2 g_1^{N/H}(x, z, Q^2) = \sum_{q, \bar{q}} e_q^2 (\Delta) q(x, Q^2) (\Delta) D_q^H(z, Q^2) , \quad (13)$$

the position of the Δ depending on which particle is polarized. The corresponding NLO corrections can be found in ^{14,15,16,1}.

The most interesting observable at HERA with respect to the determination of the polarized Λ fragmentation functions is of course the asymmetry for the production of *polarized* Λ 's from an unpolarized proton, defined by $A^\Lambda \equiv g_1^{p/\Lambda}/F_1^{p/\Lambda}$. Such a measurement would be particularly suited to improve the information on polarized fragmentation functions. In fig. 3a), we show our LO and NLO predictions for HERA with polarized electrons and *unpolarized* protons using the GRV parton distributions ⁶, integrated over the measurable range $0.1 \leq z \leq 1$. Good perturbative stability of the process is found. As can be seen, the results obtained using the three distinct scenarios for polarized fragmentation functions turn out to be completely different.

We have included in fig. 3a) also the expected statistical errors for HERA, computed assuming an integrated luminosity of 500 pb^{-1} and a realistic value of $\epsilon = 0.1$ for the efficiency of Λ detection. Comparing the asymmetries and the error bars in fig. 3a) one concludes that a measurement of A^Λ at small x would allow a discrimination between different conceivable scenarios for polarized fragmentation functions. Fig. 3b) shows our results vs. z for fixed $x = 5.6 \cdot 10^{-4}$. Again, very different asymmetries are found for the three scenarios.

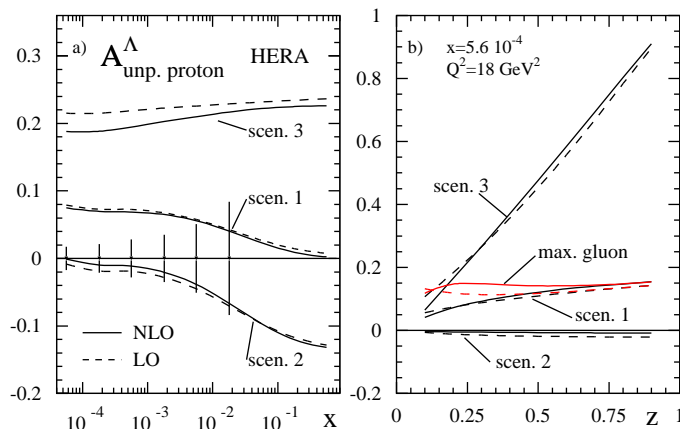


Figure 3: LO and NLO predictions for the SIDIS asymmetry for unpolarized protons and polarized Λ 's and leptons (see text) for our three distinct scenarios of polarized fragmentation functions. In a) we also show the expected statistical errors for such a measurement at HERA.

Interesting results are also obtained for fixed target experiments, where the kinematical range in x and z is complementary to the one of HERA; we refer the reader to ¹ for details on that.

The particular case of both target and hadron being polarized was originally proposed as a very good way to obtain the Δs distribution ¹⁷. The underlying assumption here was that only the fragmentation function ΔD_s^Λ is sizeable (as realized, e.g. in our scenario 1), and that therefore the only contribution to the polarized cross section has to be proportional to $\Delta s \Delta D_s^\Lambda$. In order to analyze the sensitivity of the corresponding asymmetry to Δs , we compute it using the two different GRSV sets of polarized parton densities of the proton ¹⁸, which mainly differ in the strange distribution: the so-called ‘standard’ set assumes an unbroken $SU(3)_f$ symmetric sea, whereas in the ‘valence’ scenario the sea is maximally broken and the resulting strange quark density quite small.

The results are shown in fig. 4. Unfortunately – and not unexpectedly – it turns out that the differences in the asymmetry resulting from our different models for polarized Λ fragmentation are far larger than the ones due to employing different polarized proton strange densities. In addition, a distinction between different Δs would remain elusive even if the spin-dependent Λ fragmentation functions were known to good accuracy, as can be seen from the error bars in fig. 4 which were obtained using the same parameters as before.

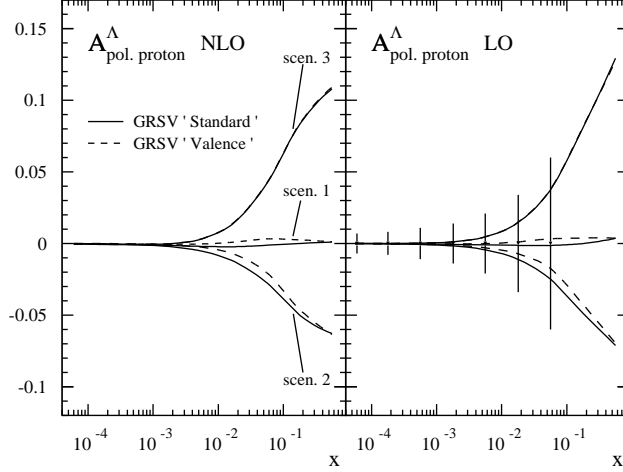


Figure 4: LO and NLO predictions for the SIDIS asymmetry for polarized protons but unpolarized leptons for two different sets of polarized parton distributions.

5 pp collisions

With the advent of RHIC, spin transfer reactions can be studied for the first time also in pp scattering at c.m.s. energies of up to $\sqrt{s} = 500$ GeV. In the following we will demonstrate that such measurements would provide a particularly clean way of discriminating between the various conceivable sets of spin-dependent Λ fragmentation functions presented above. For this purpose, only *one* polarized beam at RHIC would be needed. It should be noted here that similar (and almost equally useful) measurements could be performed also in a possibly forthcoming experiment at DESY, HERA- \vec{N} . The process we are interested in is $p\vec{p} \rightarrow \vec{\Lambda} X$ (the arrows denoting a longitudinally polarized particle) at large transverse momentum p_T of the Λ , where perturbative QCD can be safely applied. For the time being, the required partonic helicity transfer cross sections, i.e., $q\vec{q} \rightarrow q\vec{q}$, \dots , $g\vec{g} \rightarrow g\vec{g}$, are calculated only to leading order accuracy. Hence we have to restrict our analysis to LO, implying the use of LO-evolved Λ fragmentation functions.

The relevant differential polarized cross section can be schematically written as (the subscripts “+”, “-” below denote helicities)

$$\frac{d\Delta\sigma^{p\vec{p} \rightarrow \vec{\Lambda} X}}{d\eta} \equiv \frac{d\sigma^{pp+ \rightarrow \Lambda+ X}}{d\eta} - \frac{d\sigma^{pp- \rightarrow \Lambda+ X}}{d\eta} \quad (14)$$

$$= \int_{p_T^{min}} dp_T \sum_{f f' \rightarrow i X} \int dx_1 dx_2 dz f^p(x_1) \times \Delta f'^p(x_2) \times \Delta D_i^\Lambda(z) \times \frac{d\Delta\sigma^{f\bar{f}' \rightarrow iX}}{d\eta},$$

the sum running over all possible LO subprocesses, and where we have integrated over p_T , with p_T^{min} denoting some suitable lower cut-off.

The directly observable quantity will be not the cross section in (14) itself but the corresponding spin asymmetry, defined as usual by

$$A^\Lambda \equiv \frac{d\Delta\sigma^{p\bar{p} \rightarrow \bar{\Lambda} X}/d\eta}{d\sigma^{p\bar{p} \rightarrow \Lambda X}/d\eta} \quad (15)$$

where the unpolarized cross section $d\sigma^{p\bar{p} \rightarrow \Lambda X}/d\eta$ is given by an expression similar to the one in (14), with all Δ 's removed.

For the unpolarized parton distributions of the proton, f^p , appearing in (14), (15) we use the LO set of Ref. ⁶ throughout our calculations. Unless otherwise stated we use for the corresponding polarized densities Δf^p the LO GRSV “standard” scenario ¹⁸.

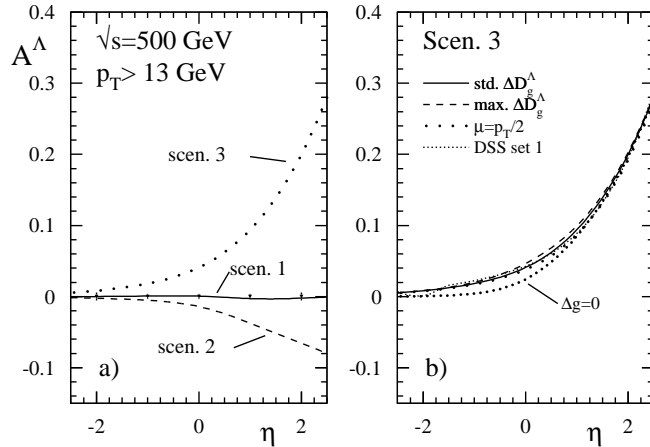


Figure 5: **(a)** The asymmetry A^Λ as a function of rapidity of the Λ at RHIC energies for the various sets of spin-dependent fragmentation functions. **(b)** same as for scenario 3 in **(a)**, but using the “maximal” ΔD_g^Λ , a hard scale $Q = p_T/2$, $\Delta g = 0$, or the DSS1 spin-dependent parton distributions

Fig. 5(a) shows our predictions for the spin asymmetry A^Λ as a function of rapidity, calculated according to Eqs. (15) and (14) for $\sqrt{s} = 500$ GeV and $p_T^{min} = 13$ GeV. Note that we have counted positive rapidity in the forward

region of the *polarized* proton. We have used the three different scenarios for the ΔD_i^Λ discussed above, employing the hard scale $Q = p_T$. The “error bars” should give an impression of the achievable statistical accuracy for such a measurement at RHIC. They have been estimated by assuming a polarization P of the proton beam of about 70%, a branching ratio $b_\Lambda \equiv BR(\Lambda \rightarrow p\pi) \simeq 0.64$, a conservative value for the Λ detection efficiency of $\epsilon_\Lambda = 0.1$, and an integrated luminosity of $\mathcal{L} = 800 \text{ pb}^{-1}$.

The behavior of A^Λ in Fig. 5(a) for the different sets of polarized Λ fragmentation functions can be easily understood from the fact that the process, in this particular kinematical region, is dominated by contributions from u and d quarks, so that the differences between the predictions in Fig. 5(a) are driven by the differences in the corresponding ΔD_u^Λ and ΔD_d^Λ . This immediately implies that the asymmetry has to be close to zero for scenario 1, negative for scenario 2 and positive and larger for scenario 3. The η -dependence is also readily understood: at negative η , the parton densities of the polarized proton are probed at small values of x_2 (i.e., in the “sea region”), where the ratio $\Delta q(x_2)/q(x_2)$ is also small. On the contrary, at large positive η , typical values of x_2 correspond to the valence region where the quarks are polarized much more strongly, resulting in an asymmetry that increases with η .

The results in Fig. 5(a) clearly demonstrate the usefulness of the proposed kind of measurements to determine the polarized Λ fragmentation functions more precisely. The expected statistical errors are much smaller than the differences in A^Λ induced by the various models. Thus an analysis of A^Λ would provide an excellent way of ruling out some of the presently allowed sets of spin-dependent Λ fragmentation functions, *provided* the observed differences in A^Λ are not obscured or washed out by the theoretical uncertainties inherent in this calculation. There are three major sources of uncertainties: the dependence of A^Λ on variations of the hard scale Q (implicit in Eq. (14)), which is of particular importance since we are limited to a LO calculation, our present inaccurate knowledge of the precise x -shape and the flavor decomposition of the polarized densities Δf^p , especially of Δg , and our ignorance of ΔD_g^Λ . Fig. 5(b) gives an example of the scale dependence of A^Λ by changing the scale from $Q = p_T$ to $Q = p_T/2$ for scenario 3. Even though $d\Delta\sigma/d\eta$ and $d\sigma/d\eta$ individually change by as much as a factor 2 at certain values of η , the uncertainty almost cancels in the ratio A^Λ . We also show in the same figure the changes in the predictions resulting from varying the polarized parton distributions, using the recent LO set 1 of Ref. ¹⁹, denoted by DSS, instead of the GRSV ¹⁸ one. As can be observed, the asymmetry remains practically unchanged, and differences can only be found at the end of phase space (at large values of η) where the cross section becomes small anyway. Also, as

an extreme way of estimating the impact of the polarized gluon distribution, we have artificially set it to zero ($\Delta g(x, \mu^2) \equiv 0$). We find that changes in our predictions only occur in the region of negative η , but are small in the interesting region $\eta > 0$ where the asymmetries are larger.

Finally, in order to examine the role played by ΔD_g^Λ in our analysis, we have used two different approaches: the standard one for our polarized fragmentation functions, where the polarized gluon fragmentation function is assumed to be vanishing at the initial scale¹ and is then built up by evolution (“std. ΔD_g^Λ ”), and a set corresponding to assuming $\Delta D_g^\Lambda \equiv D_g^\Lambda$ at the same initial scale μ (“max. ΔD_g^Λ ”) while keeping the input quark fragmentation functions unchanged. As can be observed, the resulting differences are also negligible, again due to the fact that u and d fragmentation dominate.

6 pp collisions: transverse polarization

In the case of *transverse* polarization no experimental information is available. For instance, the transversity densities, denoted by $\Delta_T q(x, Q^2)$ (or $h_1^q(x, Q^2)$), which are equally fundamental at leading twist as the $\Delta_L q(x, Q^2)$, are completely unknown for the time being. The chiral-odd $\Delta_T q(x, Q^2)$ measures the difference of the probabilities to find a quark with its spin parallel to that of a transversely polarized nucleon and of finding it oppositely polarized. Unlike the case of unpolarized and longitudinally polarized densities, there is no gluon transversity distribution at leading twist, and the $\Delta_T q(x, Q^2)$ are not accessible in inclusive DIS measurements because of their chirality properties. In a similar way, one can define transversity fragmentation functions, denoted by $\Delta_T D_q^h(x, Q^2)$, to describe the fragmentation of a transversely polarized quark into a transversely polarized hadron. In view of the promising results shown before concerning a possible measurement of the $\Delta_L D_f^\Lambda(x, Q^2)$ in $\bar{p}p \rightarrow \bar{\Lambda}X$, it seems worthwhile to study this reaction for the situation of transverse polarization at RHIC, i.e., for $p^\uparrow p \rightarrow \Lambda^\uparrow X$. In order to be able to make sensible predictions for the possible spin-transfer asymmetries for this process, we will exploit the positivity constraints derived in²⁰ to constrain the involved quantities $\Delta_T q(x, Q^2)$ and $\Delta_T D_q^h(x, Q^2)$ in a non-trivial way. Let us first recall that a positivity constraint at the naive parton model level was obtained for the $\Delta_T q(x)$, which reads²⁰

$$2|\Delta_T q(x)| \leq q(x) + \Delta_L q(x) . \quad (16)$$

An analogous positivity bound for the fragmentation functions of a quark q into a hadron h holds, namely

$$2|\Delta_T D_q^h(x)| \leq D_q^h(x) + \Delta_L D_q^h(x) . \quad (17)$$

This new result is maintained by the QCD Q^2 evolution at leading order (LO). We will use these non-trivial bounds (16) and (17) to constrain the unmeasured transversity parton densities $\Delta_T q(x, Q^2)$ and fragmentation functions $\Delta_T D_q^h(x, Q^2)$ in our studies of the spin-transfer asymmetry for transversely polarized Λ baryon production at RHIC below.

The spin-transfer cross sections for the subprocesses $ff' \rightarrow iX'$ have been known for quite some time. The cross sections for the transversity case were presented in ²¹ in a form that also allows us to distinguish between the situations “ S ” and “ N ”, i.e., when the final-state particle “ i ”, and hence the Λ , is transversely polarized in (S), or normal (N) to, the scattering plane.

Before we can estimate the spin-transfer asymmetry (which is usually called D_{NN}^Λ) for transversely polarized Λ baryon production, we have to specify the various different parton distributions and fragmentation functions involved in this calculation. We will use the approach of *saturating* the positivity inequalities given in Eqs. (16) and (17) at the input resolution scale μ to constrain the unknown transversity parton densities $\Delta_T q(x, Q^2)$ and the Λ fragmentation functions $\Delta_T D_q^\Lambda(x, Q^2)$, respectively. Such a framework is sufficient to derive a more or less rigorous estimate for an *upper bound* for the expected spin-transfer asymmetry D_{NN}^Λ . Since all relevant helicity transfer subprocess cross sections are available only at the Born level, we restrict ourselves also to LO for the Q^2 evolutions of the involved parton density and fragmentation functions. More precisely, for the $\Delta_T q(x, Q^2)$ we use the unpolarized GRV ⁶ and the longitudinally polarized GRSV ¹⁸ LO parton densities q and $\Delta_L q$, respectively, on the r.h.s. of Eq. (16). The unpolarized and the longitudinally polarized fragmentation functions D_q^Λ and $\Delta_L D_q^\Lambda$ serve to constrain the transversity fragmentation functions $\Delta_T D_q^\Lambda(x, Q^2)$ via the bound in (17).

Fig. 6 shows our predictions for the spin-transfer asymmetry D_{NN}^Λ as a function of rapidity for $\sqrt{s} = 500$ GeV and $p_T^{min} = 13$ GeV. We have used the three different scenarios for the $\Delta_T D_q^\Lambda$ discussed above, employing the hard scale $Q = p_T$. The possibility to have negative and positive asymmetries of the same size for each scenario reflects the freedom in the choice of the sign for the $\Delta_T D_q^\Lambda$ and the $\Delta_T q$ in Eqs. (17) and (16), respectively. The “error bars” in Fig. 6 should give again an impression of the achievable statistical accuracy for such a measurement at RHIC. They have been estimated by using the same parameters as in the longitudinal case. In Fig. 6 we have also studied the impact of one of the major theoretical uncertainties in a LO calculation of D_{NN}^Λ , the dependence on variations of the a priori unknown hard scale Q in (14). Luckily, it turns out that D_{NN}^Λ depends only very weakly on the value of the hard scale in the range $Q = p_T/2$ to $Q = 2p_T$, as is demonstrated for scenario 3 in Fig. 6 (very similar results hold for the other two scenarios).

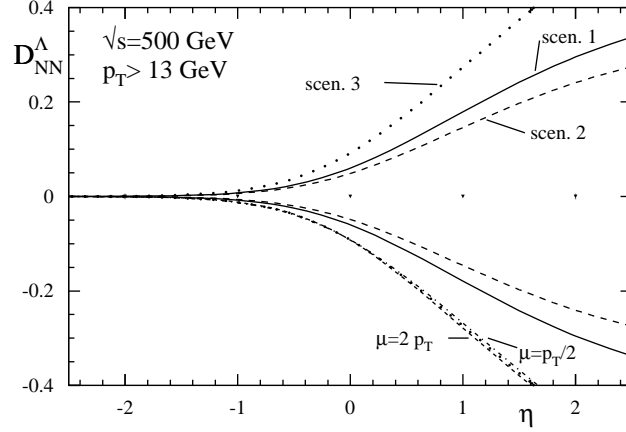


Figure 6: Upper bounds for the spin-transfer asymmetry D_{NN}^{Λ} as functions of the rapidity of the produced Λ at RHIC energies. For “scenario 3” we also illustrate the typical theoretical uncertainty induced by a variation of the hard scale Q in (14) in the range $p_T/2$ to $2p_T$.

The results shown in Fig. 6 clearly demonstrate the usefulness of studying also the production of transversely polarized Λ hyperons at RHIC. Of course, one should keep in mind that the asymmetries presented in Fig. 2 represent only a rough *upper bound* of what can be expected in an actual measurement. Hence the measured asymmetry will possibly be considerably smaller with respect to our prediction, but even when reduced by a factor of 2 or 4, a measurement of D_{NN}^{Λ} would still remain feasible since the expected statistical errors are very small.

7 Conclusions

Summarizing, we have shown that polarized fragmentation functions for Λ baryons are not well determined by the available LEP data. In order to quantify the uncertainties on them we have introduced 3 different scenarios, which are particularly useful to study the sensitivity of difference observables on them. We have first analyzed the case of Semi-Inclusive DIS, showing that measurements with unpolarized protons would allow to do a flavor decomposition of the fragmentation functions. Furthermore, it has been shown that, counting with a polarized proton target or beam, the extraction of Δs in the proton is not feasible from Λ measurements, contrary to what has been claimed. In the case of proton-proton collisions, we have shown that there are very good

prospects to measure both longitudinally and transversely polarized Λ fragmentation functions at RHIC.

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References

1. D. de Florian, M. Stratmann and W. Vogelsang, *Phys. Rev. D* **57**, 5811 (1998).
2. D. de Florian, M. Stratmann, and W. Vogelsang, *Phys. Rev. Lett.* **81**, 530 (1998).
3. D. de Florian, J. Soffer, M. Stratmann, and W. Vogelsang, *Phys. Lett. B* **439**, 176 (1998).
4. G. Altarelli, R.K. Ellis, G. Martinelli and S.Y. Pi, *Nucl. Phys. B* **160**, 301 (1979).
5. For a compilation of published e^+e^- data see: G.D. Lafferty, P.I. Reeves, and M.R. Whalley, *J. Phys. G* **21**, A1 (1995).
6. M. Glück, E. Reya and A. Vogt, *Z. Phys. C* **67**, 433 (1995).
7. V. Ravindran, *Nucl. Phys. B* **490**, 272 (1997).
8. M. Stratmann and W. Vogelsang, *Nucl. Phys. B* **496**, 41 (1997).
9. M. Burkardt and R.L. Jaffe, *Phys. Rev. Lett.* **70**, 2537 (1993).
10. R.L. Jaffe, *Phys. Rev. D* **54**, 6581 (1996).
11. A. Bravar, A. Kotzinian, and D. von Harrach, *Eur. Phys. J. C* **2**, 329 (1998).
12. B.-Q. Ma and J. Soffer, *Phys. Rev. Lett.* **82**, 2250 (1999).
13. D. Graudenz, *Nucl. Phys. B* **432**, 351 (1994).
14. D. de Florian, C. García Canal and R. Sassot, *Nucl. Phys. B* **470**, 195 (1996).
15. D. de Florian and R. Sassot, *Nucl. Phys. B* **488**, 367 (1997).
16. M. Stratmann, Ph.D. Thesis, Univ. Dortmund report DO-TH 96/24.
17. W. Lu and B.-Q. Ma, *Phys. Lett. B* **357**, 419 (1995).
18. M. Glück, E. Reya, M. Stratmann and W. Vogelsang, *Phys. Rev. D* **53**, 4775 (1996).
19. D. de Florian, O. Sampayo and R. Sassot, *Phys. Rev. D* **57**, 5803 (1998).
20. J. Soffer, *Phys. Rev. Lett.* **74**, 1292 (1995).
21. M. Stratmann and W. Vogelsang, *Phys. Lett. B* **295**, 277 (1992).